HEAT TRANSFER MECHANISM IN AN ELECTRIC ARC MOVING THROUGH A MAGNETIC FIELD

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A model of an electric arc moving through a magnetic field is analyzed. Numerical estimates and universal characteristics indicate that the basic mechanism of heat removal from the arc column is the transmission of energy by the gas stream flowing inside the arc.

A gaseous discharge moves through a magnetic field due to the primary action of this field on moving electric charges. For this reason, scientists have constructed the model of an arc in a magnetic field approximately as shown in Fig. 1. Charged particles are accelerated by the magnetic field, collide with neutral particles, and impart to the latter a part of their energy. As a result, the ionized gas begins to flow in the direction $[\vec{j}\vec{B}]$. At the arc boundary the hot gas stream encounters much denser cold gas, splits into two streams, and returns along the outer column boundary while it loses its energy by convective heat transfer to the ambient gas around the arc [1]. Furthermore, some part of the Joule heat generated in the arc is lost by conductive and radiative heat removal. It is still not clear which of these heat transfer mechanisms play the major role in determining the arc characteristics. Inasmuch as electrons have a higher mobility than ions, they break away from the latter during their motion and accumulate in the front region, where they form a negative space charge. The electric field intensity between this negative space charge and the positive column decelerates the electrons and accelerates the ions so as to equalize, under steady-state conditions, the velocities of both kinds of charges.

The intensity of the thus generated electric field can, in the case of singly charged ions of one polarity, be expressed as

$$E_{\mathrm{Hall}} \approx w_e B. \tag{1}$$

This intensity can be of the same order of magnitude as the intensity of the external electric field. At T = 12,000°K, for example, $E_{\mathrm{Hall}}/E = \sigma B/en_e = 0.48$ B and at T = 9000°K (at the periphery of the column) $E_{\mathrm{Hall}}/E = 2.65$ B. The magnetic field in actual devices is usually B = 0.2–0.5 T.

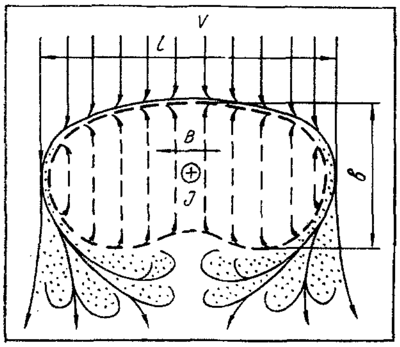

Fig. 1. Flow of gas inside the column of an electric arc moving through a magnetic field.

The Hall intensity of the electric field can cause the arc column to slant [1].

An important role in establishing the arc velocity relative to the ambient medium plays the shape of the column cross section. It has been shown in [2, 3] that an arc column "flattens" under the action of opposing electromagnetic and gasodynamic forces. The arc dimension parallel to the external magnetic field (column width) is larger than the dimension parallel to the direction of gas flow (column thickness). For a simple evaluation of the amount of energy removed from an arc by a gas stream, we approximate the column section by a rectangle (Fig. 2).

The temperature inside an arc column is of the order of 10^4 °K. In [4], for example, the temperature in an arc moved through

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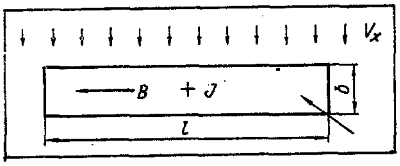

Fig. 2

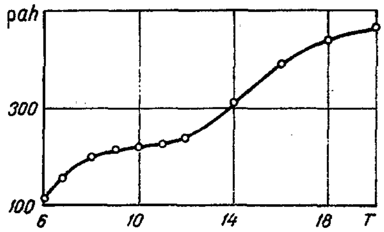

Fig. 3

Fig. 2. Simplified geometry of an arc column blasted transversely.

Fig. 3. Group $\rho a h$ (kW/cm^2) as a function of the air temperature T (°K) under atmospheric pressure.

air by a magnetic field between parallel electrodes was T = (11.9-12.9) · 10^3 °K. The density of plasma in the arc column was 1.32 · 10^{-2} kg/m^3 at T = 12 · 10^3 °K and under atmospheric pressure [5], while the density of the ambient atmosphere was 1.18 kg/m^3 at P = 1 atm and T = 300°K. The difference in densities is here almost two orders of magnitude. For this reason, only a negligibly small part of the blowing gas can pass through the column and all the remaining stream must flow around the arc as if the latter were a solid body.

In this case an aerodynamic pressure head $\rho_X V_X^2/2$ acts on the arc, balanced by the pressure of the hot gas stream inside and partly by the electromagnetic force $[\vec{J}\vec{B}]$; the braking effect of the electromagnetic force is negligible, to the first approximation, because the thickness of the arc column (across which the inner stream is accelerated) is larger than the dimension of the boundary region along which the streams collide and the temperature is higher inside the column. In coordinates referred to the arc column we then have

$$\frac{\rho_x V_x^2}{2} \approx \frac{\rho_G V_G^2}{2}. \qquad (2)$$

An analogous result has been obtained in [1].

It follows from here that the maximum velocity at the front boundary of the hot stream V_G should be higher by one order of magnitude than the velocity of the external blast V_X. It appears, moreover, that the Mach number is approximately the same for both the cold and the hot stream, inasmuch as the velocity of sound is approximately inversely proportional to the square root of the gas density:

$$a \approx \sqrt{\gamma \frac{P}{\rho}}. \qquad (3)$$

If we disregard the weak variation in γ with rising temperature, and consider that $P_G \approx P_X$, then we have from (2) and (3)

$$M_G \approx M_x. \qquad (4)$$

The gas velocity inside the column is rather high, even at moderate arc velocities, so that most of the power released in the arc can be removed by the gas stream. This makes the "through-blast" model of an arc acceptable. For such a simplified model we then have the balance of forces

$$C \frac{\rho_x V_x^2}{2} l = IB \qquad (5)$$

and the balance of power

$$\rho_G a_G h_G M l = \frac{I^2}{\sigma_G b l}. \qquad (6)$$

Equation (5) yields the width of the arc column, i.e., its dimension perpendicular to the direction of gas flow

$$l = \frac{2IB}{C \rho_x a_x^2 M^2}. \qquad (7)$$

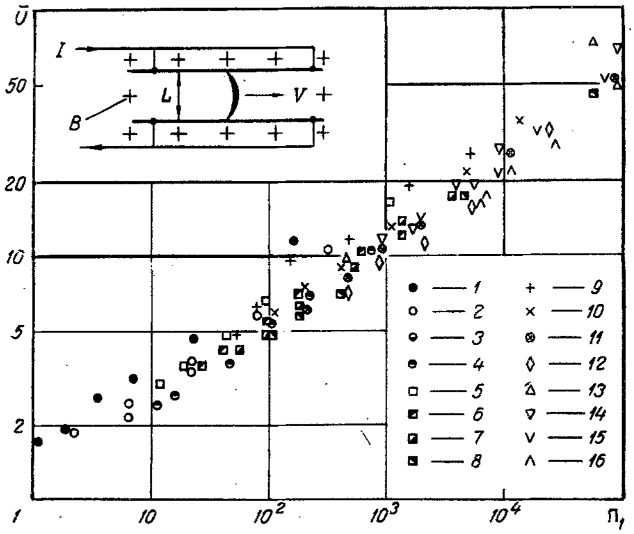

Fig. 4. Relation between dimensionless numbers $\bar{U} = UL\sigma_0/I$ and $\Pi_1 = \rho_0\sigma_0^2h_0^2BL^5/I^3$ for an arc moving through an external magnetic field between parallel copper electrodes in air. L = 0.0127 m: 1) B = 0.012 T; 2) 0.025 T; 3) 0.0535 T; 4) 0.108 T. L = 0.0191 m: 5) B = 0.0125 T; 6) 0.0265 T; 7) 0.0535 T; 8) 0.108 T. L = 0.0245 m: 9) B = 0.0125 T; 10) 0.027 T; 11) 0.054 T; 12) 0.108 T.

The thickness b can be determined from (5) and (6):

$$b = \frac{C^2\rho_x^2 a_x^4 M^3}{4B^2\sigma_G \rho_G a_G h_G}. \qquad (8)$$

At a given pressure and for a given kind of the ambient gas, the group $\sigma_G \rho_G a_G h_G$ is a function of the arc temperature only:

$$f(T) = \sigma_G \rho_G a_G h_G. \qquad (9)$$

According to (8), dimension b does not depend on the arc current but is strongly related to the blast velocity, the magnetic induction, and the arc temperature. With the blast velocity, the magnetic induction, and the arc current given, however, one equation is not sufficient for determining b and T. One may then apply the minimum principle:

$$\frac{\partial E}{\partial T} = 0. \qquad (10)$$

Ohm's law $J = \sigma E$ yields for our model

$$E = -\frac{2B}{C\rho_x a_x^2 M} f_1(T), \qquad (11)$$

where

$$f_1(T) = \rho_G a_G h_G \qquad (12)$$

represents the thermal flux density due to the gas stream inside the arc column at Ma = 1.

If function $f_1(T)$ has a minimum, then in our simplified model the arc temperature depends on the kind of blasting gas and on its pressure. The blast velocity, the arc current, and the magnetic field should not affect the arc temperature. This conclusion has been confirmed by experimental evidence. A current change from 500 A to 1800 A in [4], for instance, did not affect the $(12.4 \pm 0.5) \cdot 10^3$ °K temperature of the air-discharge under atmospheric pressure.

Function $f_1(T)$ for air under atmospheric pressure is shown in Fig. 3. According to the diagram, the curve of thermal flux density has no minimum. Within the $(9\text{–}11) \cdot 10^3$ °K temperature range, on the other hand, the curve is almost horizontal. If our model of a blasted arc is valid, then the temperature should stabilize immediately after the plateau range of the curve, inasmuch as power removal from the arc by means of a gas stream under further rising temperature would require a decrease in the Mach number. Thus, calculations and measurements seem to agree.

Let us determine the dimensions of the arc column of rectangular cross section under the test conditions in [4], for example: $I = 1000$ A, $B = 0.086$ T, $Ma = 0.316$, $P = 1$ atm, $\rho_x = 1.18$ kg/m^3. We assume a temperature $T = 1.2 \cdot 10^4$ °K and $C = 1$. Then $f(T) = 1.6 \cdot 10^{-3}$ A^2/m^3. From (8) we obtain $b = 1.3$ mm, and l according to (7) is 10.9 mm. The ratio $l/b = 8.4$.

This calculation agrees qualitatively with the data in [2, 3], but not with the arc measurements in [4]. The relation between the dimensions there was reverse: the column thickness b was much larger than the column width l. With the two-dimensionality of the gas flow and with other modes of heat transfer than "blasting" the column taken into account, a smaller $\sigma_{mean}/\sigma_{max}$ ratio will certainly yield a smaller l/b ratio than calculated earlier. With the gas stream playing the major role in removing heat from the arc column, however, it is difficult to expect the ratio l/b to be smaller than unity. The indirect measurement of the arc thickness on the basis of spectrographic data in [4] did, evidently, include the length of the electrically almost nonconducting high-temperature gas "tail."

The velocity of the gas stream inside the arc column is rather high. In the earlier example, for instance, $V_G = 1180$ m/sec and $a_G = 3739$ m/sec at $Ma = 0.316$ and $T = 12{,}000$°K. The thermal flux density there was $\rho Vh = 76$ kW/cm^2.

These estimates indicate that, even a very roughly approximate model based on a gas stream as the major means of removing heat from the arc column yields results which agree fairly well with experiments. In order to generalize the volt–ampere characteristic of an arc, therefore, it is necessary to introduce a criterial "blast" number as a basic parameter:

$$\Pi_1 = \frac{\rho_0 h_0^2 \sigma_0^2 B L^5}{I^3}, \tag{13}$$

for the case of an arc moving through an external magnetic field B [6].

For an arc moving through its intrinsic magnetic field we have instead of Π_1 [6]:

$$\Pi_2 = \frac{\rho_0 \mu_0 h_0^2 \sigma_0^2 L^4}{I^2}. \tag{14}$$

In Fig. 4 is shown the universal volt–ampere characteristic of an arc moving through an external magnetic field $B = 0.012$–0.108 T between parallel copper electrodes $L = 0.0127$–0.038 m long [7]. The ambient medium here is air at $P = 1$ atm and the arc current (I) varies from 120 A to 1000 A. According to the graph, the use of one criterial number Π_1 yields a satisfactory generalization.

We have also tried to generalize on the basis of other dimensionless groups characterizing various modes of heat removal from the arc column. The following power-law approximation was used as reference:

$$\lg A = \alpha + \beta \lg B. \tag{15}$$

The values of α, β, and the standard deviation Δ for various hypothetical modes of heat transfer are listed in Table 1.

According to Table 1, the scatter of test points about the universal volt–ampere characteristic is least with the Π_1 number (row 2) as the criterial parameter. The gas acceleration (row 4) has almost no effect on the arc characteristic, and attempts to generalize on the basis of this criterial process alone have resulted in a larger standard deviation than the standard deviation from the ungeneralized characteristic. Next to the generalization based on "blast" removal of heat from the arc column, the generalization based on conductive heat removal (row 3) yields less scatter of test points than the ungeneralized characteristic. The energy characteristics of the arc column are almost unaffected by convective heat transfer at its surface (row 6) and the generalization based on this mode of heat removal yields more scatter of test points.

Of special interest is row 5 representing the generalization of the volt–ampere characteristic based on the criterial blast number Π_2, the Π_2 number reflecting the effect of the intrinsic magnetic field of the arc rather than the effect of the external magnetic field reflected in the Π_1 number. The magnitude of the

TABLE 1. Values of Coefficient α, Exponent β, and Standard Deviation Δ for Generalizing the Volt–Ampere and the Velocity Characteristics of an Arc Blasted Transversely in Various Modes

No.	A	α	B	β	Δ
1	$\bar{U}$	142,8	I	–0,096	0,270
2	$\frac{UL\sigma_0}{I}$	1,39	$\frac{\sigma_0^2\rho_0 h_0^2 L^5 B}{I^3}$	0,32	0,154
3	$\frac{UL\sigma_0}{I}$	50	$\frac{\sigma_0\lambda_0 T_0 L^2}{I^2}$	0,63	0,210
4	$\frac{UL\sigma_0}{I}$	0,5	$\frac{\rho_0 h_0 L}{IB}$	0,43	0,752
5	$\frac{UL\sigma_0}{I}$	415,83	$\frac{\sigma_0^2\rho_0 h_0^2\mu_0 L^4}{I^2}$	0,45	0,190
6	$\frac{UL\sigma_0}{I}$	8,64	$\frac{\rho_0 h_0^2 IBL}{\lambda_0^2 T_0^2}$	0,012	0,881
7	$\frac{\rho_0 V^2 L}{IB}$	0,08	—	—	0,332
8	$\frac{\rho_0 V^2 L}{IB}$	0,043	$\frac{\sigma_0^2\rho_0 h_0^2 BL^5}{I^3}$	0,07	0,210
9	$\frac{\rho_0 V^2 L}{IB}$	0,103	$\frac{\sigma_0\lambda_0 T_0 L^2}{I^2}$	0,16	0,176
10	$\frac{\rho_0 V^2 L}{IB}$	0,024	$\frac{\rho_0 h_0 L}{IB}$	0,024	0,240
11	$\frac{\rho_0 V^2 L}{IB}$	0,16	$\frac{\sigma_0^2\rho_0 h_0^2\mu_0 L^4}{I^2}$	0,102	0,200
12	$\frac{\rho_0 V^2 L}{IB}$	0,169	$\frac{\rho_0 h_0^2 IBL}{\lambda_0^2 T_0^2}$	0,008	0,270

standard deviation indicates that the influence of the intrinsic magnetic field can be rather substantial, despite the special measures taken to eliminate its effect during the experiment (the current leads were connected on opposite sides of each electrode). Deflections of the arc column may possibly produce sufficiently strong forces – comparable in magnitude with the force between the arc current and the external magnetic field.

The method of generalization is certainly not a sufficient basis for drawing final conclusions, inasmuch as any criterial parameter can to some degree inadvertently account for the effects of most diverse factors. The effect of any one process can be accounted for reliably only if all its different manifestations concur. Nevertheless, the significance of the criterial blast number based on the motion of the arc column through its intrinsic magnetic field must not be disregarded.

The criterial blast parameters are significant also in the universal velocity characteristics of an arc. Of all heat transfer processes, heat removal by conduction is most significant here. It seems that the width of the arc column depends strongly on it, because the gas stream does not remove any heat in the direction perpendicular to the flow.

Here again an anomaly is noted in the effect of the criterial blast parameter which reflects the influence of the intrinsic magnetic field. The generalization of the volt–ampere characteristic indicates that, in terms of energy, motion through the intrinsic magnetic field is less significant than "blast" due to the external magnetic field. But motion through the intrinsic magnetic field has a great effect on the arc velocity. This anomaly can be explained by periodic deflections of the arc column perpendicularly to the direction of motion.

At this time it is still difficult to make a definite assessment as to the role which the instability of the arc shape plays in determining its energy and velocity characteristics. If this instability is significant, however, then the theoretical methods of designing transversely blasted arcs will become much more complex.

Table 1 shows that least significant of all energy processes here is convective heat transfer at the surface of the arc column (rows 6 and 12). The corresponding Peclet number as the criterial parameter makes a generalization of the volt–ampere characteristic less accurate and contributes very little to a generalization of the velocity characteristic. It must be assumed that the Peclet number degenerates as a result of turbulence in the stream at the periphery of the arc column.

NOTATION

B is the magnetic induction;
C is the aerodynamic drag coefficient;
E is the electric field intensity;
I is the arc current;
L is the characteristic dimension;
Ma is the Mach number;
P is the pressure;
T is the temperature;
V is the velocity;
w is the electron drift velocity;
a is the velocity of sound;
b is the arc thickness (dimension along the direction of arc motion);
l is the arc width (dimension perpendicular to the direction of motion);
J is the current density;
γ is the ratio of specific heats;
λ is the thermal conductivity;
μ is the magnetic permeability of the vacuum;
ρ is the density;
σ is the electrical conductivity.

Subscripts

G denotes hot (gas);
e denotes electron;
o denotes governing;
x denotes cold.

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